Scaling and universality in heart rate variability distributions

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Abstract

We find that a universal homogeneous scaling form describes the distributions of cardiac variations for a group of healthy subjects, which is stable over a wide range of time scales. However, a similar scaling function does not exist for a group with a common cardiopulmonary instability associated with sleep apnea. Subtle differences in the distributions for the day- and night-phase dynamics for healthy subjects are detected. © 1998 Elsevier Science B.V. All rights reserved.

Time series of beat-to-beat (RR) heart rate intervals obtained from digitized electrocardiograms are known to be nonstationary and exhibit extremely complex behavior [1,2]. A typical feature of such nonstationary signals is the presence of "patchy" patterns which change over time. Nonstationarity, an important aspect of biological variability, can be associated with regimes of different drifts in the mean value of a given signal, or with changes in its variance which may be gradual or abrupt. Heterogeneous properties may be even more strongly expressed in certain cases of abnormal heart activity.

Differences between healthy and abnormal cardiac dynamics are known to be reflected in different correlations and power spectra [3–5]. However, it is currently widely assumed [6,7] that the difference in time series of interbeat intervals in sick and healthy adults *lies not in the distribution of the interbeat variations but rather in their time ordering*. It has been also hypothesized that even if the interbeat variations are different (e.g. smaller) during illness, the pattern of heart rate variability might be otherwise very similar to that during health. In such a case, the interbeat variations for normal and abnormal cardiac dynamics, once normalized, would have the same distribution. Such assumptions are based on more conventional studies of interbeat intervals or increments which essentially amount to taking derivatives of the heart rate

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signal and thus extracting pointwise characteristics. Here we advocate the idea that:
(i) *different dynamical patterns can be observed on different time scales* correspond-
ing to the actual time scales of the underlying physiological processes; (ii) masking
effects of nonstationarities have to be properly reduced.

We begin by taking a subset of healthy subjects and analyze the distribution of
interbeat intervals during night. After subtracting the global average, and rescaling the
distributions while preserving the normalization to unit area, we find that the data do
not collapse onto a single curve (Fig. 1). Moreover, we find no difference between
these histograms for the healthy subjects and a group with cardiopulmonary disorder
caused by sleep apnea (Fig. 2). This test clearly demonstrates how nonstationarities
affect data and the necessity to reduce nonstationarities in order to observe hidden
scaling behavior.

To analyze the properties of human cardiac activity we use a method which we call
"cumulative variation amplitude analysis", designed to address nonstationary behavior
[8]. This method comprises sequential application of a set of algorithms based on
wavelet and Hilbert transform analysis.

The wavelet transform [9] is defined as a convolution of a signal with an analysing
wavelet and is sometimes called a "mathematical microscope" because it allows one to
study properties of the signal on any chosen scale. The wavelet transform allows one to
focus ("extract") from the data particular features. Since the object of our study is the
variations in the heart rate signal, we choose as analyzing wavelets derivatives of the
Gaussian function, which allow us to extract these variations [10]. One can argue that
the same can be done by simply subtracting consecutive interbeat intervals, but such
standard analysis does *not* distinguish healthy from unhealthy cardiac dynamics [11].
The reason is the wavelet transform in addition to extracting the cumulative variations
in the heart rate signal over given time scale, reduces masking effects of the nonsta-
tionarities, since the analyzing wavelet used is orthogonal to local polynomial trends.

The next step of the "cumulative variation amplitude analysis" is to extract the
amplitudes of the variations in the beat-to-beat signal by means of an analytic signal
approach (Hilbert transform) [12] which also does not require stationarity. The use
of Hilbert transform provides for a statistics reflecting the *duration* of segments with
different amplitudes of variations in the wavelet-transformed signal.

We studied the distribution of the amplitudes of the beat-to-beat variations for a
group of healthy subjects ($N = 18$: 5 males and 13 females; age: 20–50, mean –
34) and a group of subjects with obstructive sleep apnea [13] ($N = 16$ males; age:
32–56, mean – 43). To minimize nonstationarity due to changes in the level of activ-
ity, we begin by considering night phase (12 pm–6 am) records of interbeat intervals
($\approx 10^4$ beats) for both groups. Inspection of the distribution functions of the amplitudes
of the cumulative variations reveals marked differences between individuals (Fig. 2a
in Ref. [8]). These differences are not surprising, given the underlying physiologi-
cal differences among healthy subjects. To test the hypothesis that there is a hidden,
possibly universal structure to these heterogeneous time series, we rescale the dis-
tributions and find for all healthy subjects that the data conform to a single scaled
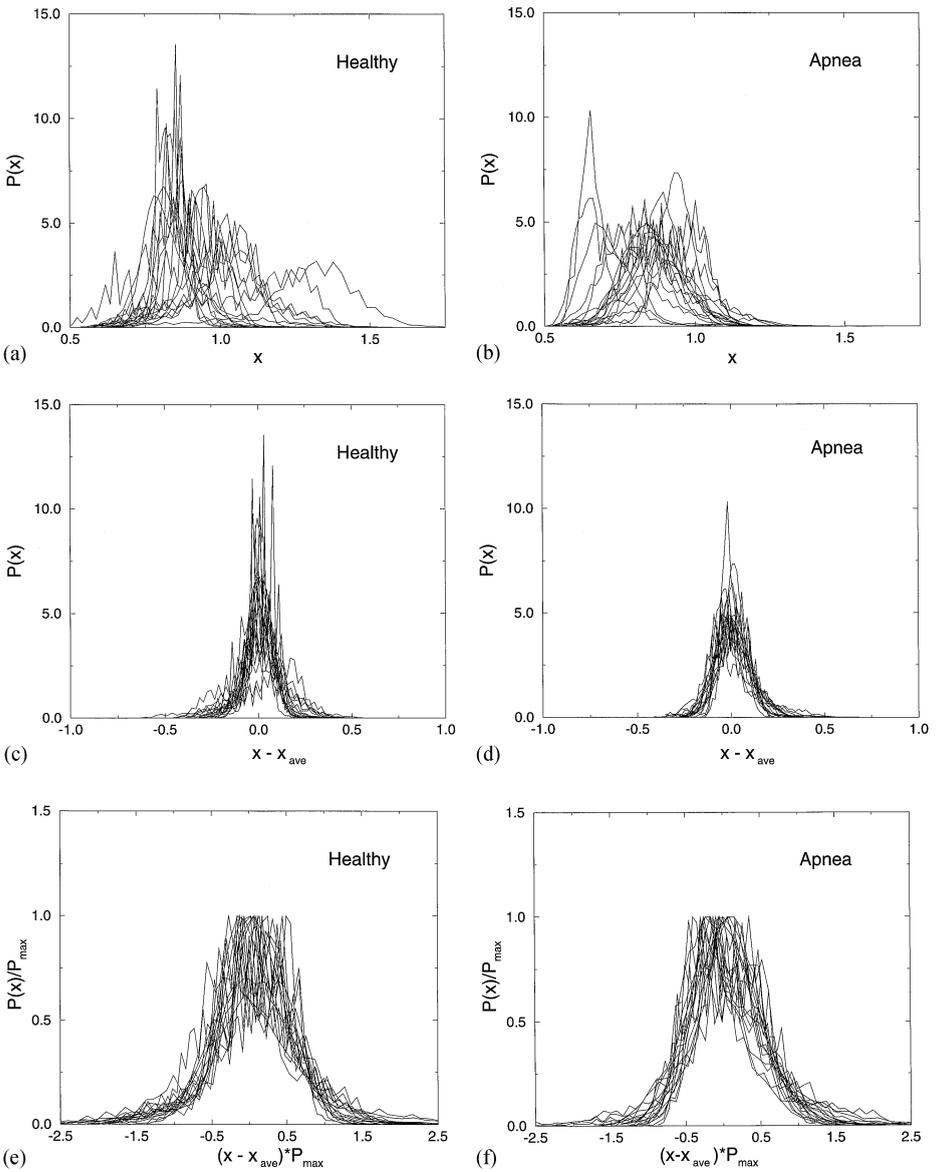
Fig. 1. Distributions of the interbeat intervals, where x denotes the interbeat interval time (a) for a random subset of nine healthy subjects and (b) nine subjects with sleep apnea. The same distributions are presented after subtracting the global average in (c) and (d). No data collapse is observed after rescaling of these distributions for the healthy (e) and the apnea (f) subjects. Compare with Fig. 3.

plot ("*data collapse*") (Fig. 2b in Ref. [8]). We find the rescaled data is well fit with a homogeneous gamma distribution, defined with a single parameter. Such behavior is reminiscent of a wide class of well-studied physical systems with universal scaling properties [14,15]. In contrast, the subjects with *sleep apnea* show individual
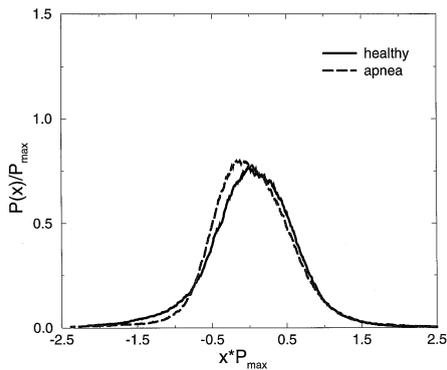
Fig. 2. Averaged distributions (from Fig. 1c and Fig. 1f) of the interbeat intervals are identical for the healthy and apnea group.
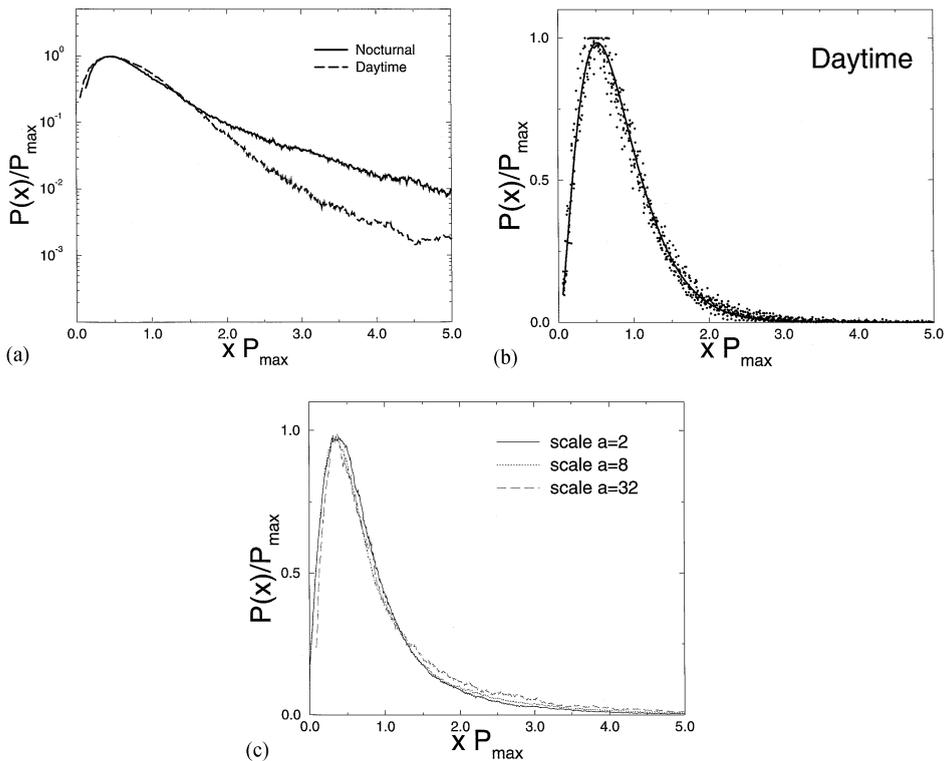

Fig. 3. (a) Plots of the day- and night-phase distributions. Data is averaged over a subset of 18 healthy subjects. (b) The solid line is an analytic fit of the rescaled distributions of the beat-to-beat variation amplitudes of 18 healthy subjects during *day* hours to a Gamma distribution with $v = 1.8 \pm 0.1$, thereby showing that the observed common structure for the healthy heart dynamics is not confined to the nocturnal phase. (c) Group average of the rescaled distributions of the cumulative variation amplitudes for the healthy individuals during *nocturnal* hours. Note that the observed Gamma scaling is *stable* for a wide range of the wavelet transform scale.
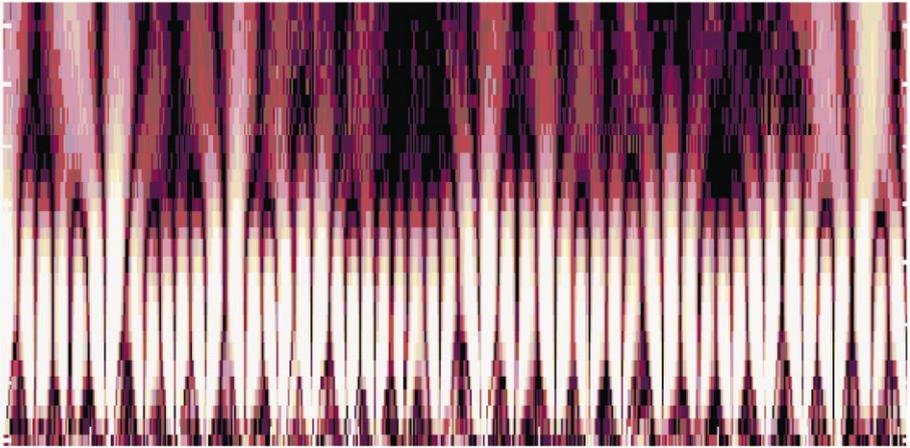
(A)
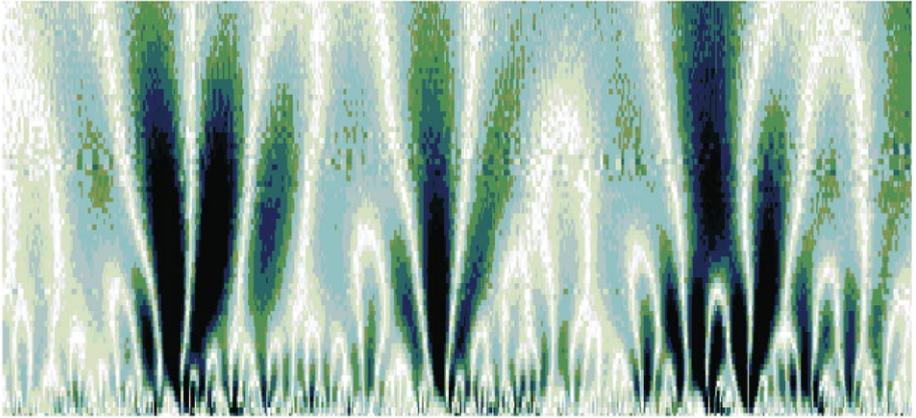
(B)

Fig. 4. Color-coded wavelet analysis of *RR* signals. The *x*-axis represents time (≈ 2000 beats) and the *y*-axis indicates the scale of the wavelet used ($a = 1, 2, \ldots, 60$) with large scales at the top. The brighter colors indicate larger values of the wavelet amplitudes. The wavelet analysis uncovers a hierarchical scale invariance (Fig. 3c) and reveals a self-similar fractal structure in the healthy cardiac dynamics (A) and a loss of this fractal structure in cases with sleep apnea (B).

probability distributions which *fail* to collapse (Fig. 2d in Ref. [8]). The collapse of the individual distributions for all healthy subjects after rescaling their "individual" parameter is indicative of a "universal" structure. The term "universal" is used in the sense that a closed mathematical scaling form is established describing in a unified quantitative way the cardiac dynamics of all studied healthy subjects.

An analysis of the heart rate dynamics for healthy subjects during the daytime (noon–6 pm) indicates that the observed, apparently universal, behavior holds not only for the night phase but for the day phase as well (Fig. 3b). Semilog plots of the averaged distributions show a crossover (slower decay) in the tails of the night-phase distributions, whereas the day-phase distributions follow the exponential form over practically the entire range (Fig. 3a). Note that the tail of the observed distribution for the night phase indicates higher probability of larger variations in the healthy heart dynamics during sleep hours in comparison with the daytime dynamics.

We observe for the healthy group good data collapse with a *stable* scaling form for wavelet scales of 2 up to 64 (Fig. 3c). The stability of this scaling form (Fig. 3c) indicates that the underlying dynamical mechanisms regulating the healthy heartbeat have similar statistical properties on different time scales. Such statistical self-similarity is an important characteristic of fractal objects [3–5]. The wavelet decomposition of beat-to-beat heart rate signals can be used to provide a visual representation of this fractal structure (Fig. 4A). The wavelet transform enables us to identify self-similar patterns (arches) in these variations even when the signals change as a result of background interference. Data from sick heart (sleep apnea) lack these patterns (Fig. 4B).

In summary, we believe that the method proposed here can pick up differences that are missed by other approaches for two reasons: it can "filter out" dominant features related to nonstationarities and thereby become sensitive to hidden scaling features; and is sensitive to the time ordering of events provided a sensible choice is made for the scale parameter.

Acknowledgements

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