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Algebraically decaying noise in a system of particles with hard-core interactions*

Chung-Kang Peng[a], Shlomo Havlin,[a,b] Moshe Schwartz,[c]
H. Eugene Stanley[a] and George H. Weiss[b]

[a] *Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215, USA*
[b] *Physical Sciences Laboratory, Division of Computer Research and Technology, National Institutes of Health, Bethesda, MD 20892, USA*
[c] *Department of Physics and Astronomy, Tel-Aviv University, Ramat-Aviv, 69978 Tel-Aviv, Israel*

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We record and analyze the noise experienced by a tracer particle in a one-dimensional system of particles interacting with hard-core interactions. We find that the correlations of the noise are long-range, with an *algebraic* decay in time.

The effect of the environment of the motion of a single tracer particle is often described in terms of a friction force and a random force. The random force, termed "noise", is of necessity correlated in time since it arises from the physical evolution in time of the rest of the system. Taking into account the full physical evolution it is found, however, that these correlations are short range [1], so that the correlated physical noise can be replaced by an uncorrelated noise. Indeed, noise with short-range correlations seems to occur in any system with short-range microscopic interactions which is in equilibrium (or is allowed to approach equilibrium). However, there has been much recent interest in systems in which the motion of tracer particles can be explained only in terms of very long-range correlations present in the noise. Famous examples are the behavior of tracer particles in a turbulent fluid[#1] [6] or under the influence of a set of random velocity fields (such as a random quenched array of currents) [7, 8]. Such systems are maintained in a *non-equilibrium* state; this has a crucial

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[#1] See, e.g., the recent review [2] which includes photographs of Lévy flights and Lévy walks. See also the review [3]. For applications to turbulence and chaos, see ref. [4] and for a general discussion of the response of multivariable linear systems to Lévy fluctuations, see ref. [5].

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effect on the noise experienced by particles coupled to them even though the microscopic coupling is short-range. On the other hand, we would not expect long-range noise correlations in a system of particles interacting via a short-range two-body potential.

Here we describe an interesting and remarkably simple system that provides a counterexample to the expectations outlined above. We consider a one-dimensional hard-sphere system and find that the noise correlation is algebraically decaying in time,

$$\langle v(t)\, v(t')\rangle \propto |t-t'|^{-3/2} \qquad \text{for large } |t-t'|\,, \tag{1}$$

where $v(t)$ is the noise at time t, and $\langle v(t)\, v(t')\rangle$ is the average over many particles.

We have simulated such a system in which the rule of motion [9, 10] is that at each time step the particle has an equal a priori probability of moving one step to the right or to the left. However, once a direction has been decided the move is aborted or not, according to whether there is a nearest neighboring particle present at the intended point. The velocity of the particle can thus take on any of three possible values

$$v(t) = \begin{cases} 1 & \text{if it jumps to the right}\,, \\ 0 & \text{if it does not jump (due to a collision)}\,, \\ -1 & \text{if it jumps to the left}\,. \end{cases} \tag{2}$$

We recorded the velocity $v(t)$ of each particle over a large number (2^{14}) of time steps and Fourier analyzed the resulting record. Since we are using discrete time steps and a finite number of time steps, what we really have are Fourier *series* representing $v(t)$ as a function of $v(\omega)$ and vice versa. Fig. 1a shows $v(t)$ versus t for a *given* particle and fig. 1b its Fourier transform. The data on a single particle is, of course, very noisy. Hence we have calculated $|v(\omega)|$ for each particle; and averaged over 10000 particles to reduce fluctuations. Fig. 2 is a double logarithmic plot of $\langle |v(\omega)| \rangle$ versus ω, supporting the result

$$\langle |v(\omega)| \rangle \sim \omega^{1/4} \tag{3a}$$

for small ω. It follows that

$$F(\omega) \equiv \langle |v(\omega)|^2 \rangle \sim \omega^{1/2}\,. \tag{3b}$$

The correlation considered in (1) must be a function of the time difference,

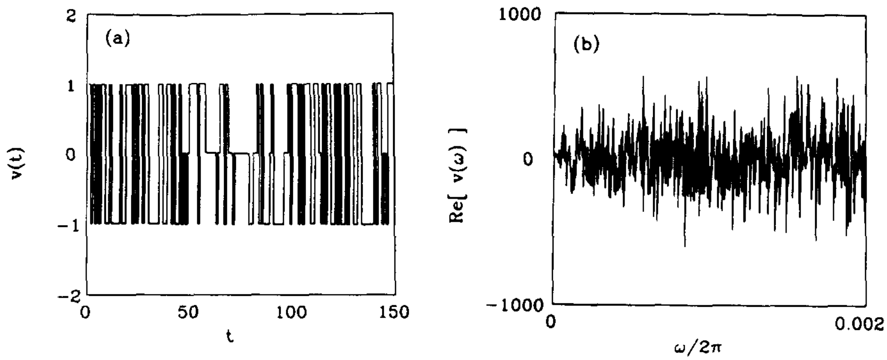

Fig. 1. One-dimensional particle diffusion with hard-core interaction (at particle density 1/4). (a) The typical velocity plot of one particle. The velocity was recorded for a very long time period (2^{14} time steps), only part of them are shown here. (b) The Fourier spectrum for the same particle.

and $F(\omega)$ is the Fourier transform of that function. From the form of $F(\omega)$ at small ω, we conclude that the functional form of the correlation at large time separation is indeed $\langle v(t)\, v(t')\rangle \propto |t-t'|^{-3/2}$.

To provide an independent check on our noise analysis, we calculate analytically $\langle \Delta x^2(t)\rangle$ for a particle that is affected by such a noise and compare it to $\langle \Delta x^2(t)\rangle$ obtained for the hard-sphere system [9, 10]. Consider a particle obeying the Langevin equation

$$\dot{x} = v(t)\,, \tag{4}$$

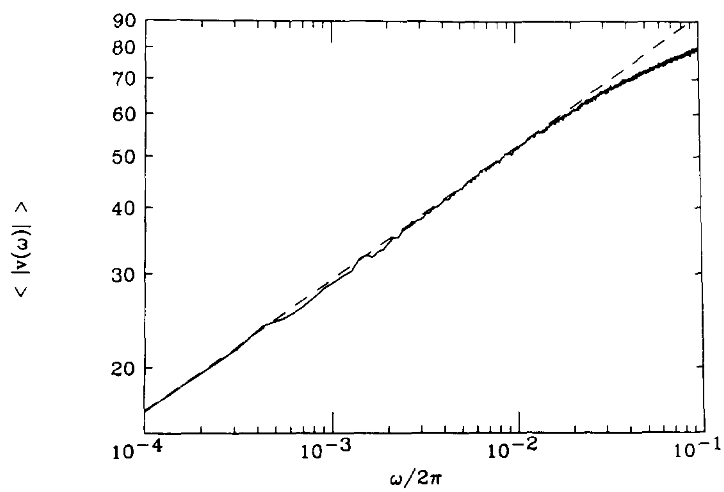

Fig. 2. Log–log plot of $\langle |v(\omega)|\rangle$ vs. ω shows a straight line for small ω. The dashed line corresponds to a slope equal to 1/4 ($\lambda = -1/2$).

where the needed statistical properties of $v(t)$ are defined in terms of its Fourier transform $v(\omega)$,

$$\langle v(\omega)\rangle = 0\,, \qquad \langle v(\omega_1)\, v(-\omega_2)\rangle = F(\omega_1)\, \delta(\omega_1 - \omega_2)\,. \tag{5}$$

The mean square displacement at time t is defined as $\langle \Delta x^2(t)\rangle \equiv \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2 \times \langle v(t_1)\, v(t_2)\rangle$. By replacing $v(t)$ with its Fourier transform we obtain

$$\begin{aligned}\langle \Delta x^2(t)\rangle &= \frac{1}{4\pi^2} \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2 \int_{-\infty}^{\infty} \mathrm{d}\omega_1 \int_{-\infty}^{\infty} \mathrm{d}\omega_2\, \mathrm{e}^{\mathrm{i}(\omega_1 t_1 - \omega_2 t_2)} \langle v(\omega_1)\, v(-\omega_2)\rangle \\ &= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \mathrm{d}\omega\, \frac{F(\omega)}{\omega^2}\, |\mathrm{e}^{\mathrm{i}\omega t} - 1|^2 \\ &= \frac{1}{\pi^2} \int_{-\infty}^{\infty} \mathrm{d}\omega\, \frac{F(\omega)}{\omega^2}\, \sin^2(\tfrac{1}{2}\omega t)\,,\end{aligned} \tag{6}$$

where we have used eq. (5). We will be interested in the case that

$$F(\omega) \sim \omega^{-\lambda} \qquad (-1 \leqslant \lambda \leqslant 1) \tag{7}$$

for small ω, which corresponds to a power law decay of the correlation function for large $t_1 - t_2$

$$\langle v(t_1)\, v(t_2)\rangle \sim |t_1 - t_2|^{-(1-\lambda)}\,. \tag{8}$$

If follows from eqs. (6) and (7) that the long-time behavior of $\langle \Delta x^2(t)\rangle$ is given by [11]

$$\langle \Delta x^2(t)\rangle \propto t^{1+\lambda}\,. \tag{9}$$

In our case $\lambda = -1/2$, so that $\langle \Delta x^2(t)\rangle$ is expected to behave as $t^{1/2}$ for large t and this is indeed the result obtained in refs. [9, 10]. We have thus an independent check on the fact that the correlations of the noise experienced by a single particle in our system decay algebraically in time in the form given by eq. (1).

It is interesting to note that in spite of the marked difference from the usual equilibrium system manifested in the effect of the system on a single particle, there is also a similarity with such systems. That similarity with equilibrium systems also distinguishes between our system and the non-equilibrium situations mentioned above as well as in diffusion on fractals [12, 13]. In those cases the algebraically decaying correlated noise is accompanied by a non-Gaussian $P(x, t)$. In the hard-core case, however, there is both analytic [9] and numerical evidence [10] (confirmed also by us) that the distribution $P(x, t)$ is in fact Gaussian, as is the case for a particle affected by uncorrelated noise. We suggest that the method of noise recording and analysis using Fourier transforms may be a useful tool in the study of complex physical systems.

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